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Faculty Working Papers

ADVERTISING: DOES THE S-CURVE APPLY?

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#72

**College of Commerce and Business Administration
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Introduction

Almost all literature on advertising agrees on one point: the effect of advertising upon sales or market shares gradually reaches a saturation point, after which additional advertising does not have any effect. Reasons for this characteristic of advertising can be found at both the individual and the aggregate level. As a person gets more and more exposure to advertising, the possibility of increasing the response (purchase, say) is limited. Larger quantities can be bought than before, but generally the response becomes a matter of purchase or no purchase (see Bogart, 1967). At the aggregate level, as the advertising increases, the additional prospects exposed are usually of lower potential than earlier prospects (see Stigler, 1961) or already exposed and committed prospects are reached (see Longman, 1971).

In the same writings on advertising, however, much discussion is often expended on the question of whether there is a "threshold" effect of advertising, that is, whether advertising will have a small effect initially and then a "takeoff" as more advertising is done. One reason for this takeoff at the individual level would be that the individual would need some repetitive stimuli before he/she acts. Against this hypothesis stands the argument that the first exposure will always be the most effective one, later exposures being less new and interesting and largely serving a supportive role. At the aggregate level the takeoff would occur because the "right" media for reaching the best prospects would be available (this argument refers most often to Network TV), and because without a certain level the advertising effort would be on the whole "drowned out" by competing messages. Against this hypothesis it has been argued that the choice of media vehicle is

such that the prospects with the highest potential are reached first anyway; and that although very small expenditures are hardly worthwhile, a small advertiser can concentrate his advertising in a few vehicles and thus make a relatively strong impact (see Bogart, 1967).

The balance of available evidence, as reviewed by Simon (1965, 1969) and Preston (1968), seems to lean towards the "no-takeoff" hypothesis. Thus, both authors when referring to sales or market shares data find very little evidence supporting the sigmoid or S-shaped curve and generally see the advertising response as exhibiting diminishing returns. On the other hand, surprisingly little empirical research has attempted a direct fit of the sigmoid to available data. Thus, no research incorporating the logistic directly is covered in Simon's two reviews or in Preston's review of scale returns. One explanation lies in the difficulty of obtaining nonexperimental data where observations occur in the increasing part of the curve. If management made their advertising decisions correctly, they would normally not stop their advertising there. This cannot provide the complete explanation, however, since a good deal of the research discussed by Simon in his two reviews covers experimental or quasi-experimental data, which presumably could be analyzed directly for a sigmoid fit. Furthermore, even if entrepreneurs would generally not like to operate on the increasing part of the curve, they might still find themselves there. This could be the case if the effectiveness of the advertising effort is partly determined by the competitive advertising input, so that what matters is the firm's "advertising share relative to competitors' shares". Then the entrepreneur would no longer be in full control of his advertising input--and increases in the spending might

very well be offset by increases in competitive spending. That this can happen is pointed out quite frequently--see, for example, Bass (1970), Bogart (1967), and Telser (1964).

Perhaps the major reason for the lack of a direct approach to the estimation of the sigmoid curve is the mathematical and statistical problems encountered. In resolving the mathematical difficulties one would generally complicate the statistical problems. Thus, the common generalized logistic function requires iterative maximum likelihood estimation techniques (see Oliver, 1966) which become quite expensive considering the type of data and the number of variables usually involved in advertising research. Conversely, the simpler estimating models are generally incapable of exhibiting the required S-shape. Accordingly, there is a need for a sigmoid function that is possible to estimate using relatively simple statistical techniques. Because such a sigmoid function has been developed (see Johansson, 1972a, and 1972b) it is now possible to estimate directly the goodness of fit of the S-curve relative to alternative simpler models of advertising effects. That is the subject of this paper.

The Data

Before dealing with the specification of the actual estimating models incorporating the sigmoid, it will be useful to have an account of the data available.

The data consist of monthly surveys of product users' preferences, and purchases (present and past) with respect to the four biggest national brands in a consumer product class (non-seasonal). Each month a different sample of 1000 respondents representative of the U.S. population was selected. Usable returns, received within the

first two weeks of mailing (later returns were eliminated because of the overlap problem), in general numbered about 600. The four national brands used in the analysis were observed during 13 consecutive months.

The surveys also provide information as to degree of usage of product, whether or not the last brand purchased was bought with a coupon, on a deal, or sale, or regular price, and what brands the respondent had bought within the last six months. In addition to the survey data, data on prices during the months observed were collected independently through general trade information and price lists.

The advertising data consist of monthly brand expenditures in four different media--Network TV, Spot TV, Magazines, and Newspapers (non-retail advertising). The observations were compiled from the records maintained by Leading National Advertisers, Inc., and, in the case of Newspapers, from the "Blue Book" published by Media Records. These data, as is well known, suffer from certain deficiencies. Not all stations in the country report the Spot TV expenditures; for the 62% that do, gross one-time rates for length of commercial and daypart, without discounts, are used, under the assumption that this will allow for the missing data. The Magazine expenditures are based upon advertising space and revenue analyses, and cover about 80% of total expenditures in consumer magazines. The Newspaper data are in linage, not dollars, and only about 75% of total newspaper advertising is covered in the Blue Books. The data for Network TV, finally, are probably the most accurate. Here expenditures refer to net time (after discount for daypart) plus talent or program estimated cost. Because of the

existence of only three network companies, these data are relatively easy to record.¹

As the frequency of purchase within the product class is on the average one purchase per month, it was deemed feasible to trace monthly effects from the media advertising inputs. Even so, it was clear that some distributed lag approach might be needed to account for a possible cumulative advertising impact over time.² It should be emphasized, however, that these data do not allow a test of whether or not the S-curve is applicable over the long run.

Because of the short time periods involved, it was decided that media inputs should be kept separated in the regression runs. This would allow different coefficients and different lags to emerge-- although in the long run possible media differences tend to disappear, in the short run they are often assumed to exist (see, for example, Bogart, 1967). The total advertising impact is then measured as the sum of the different effects over time and over media.

Model Specification and Estimation Method

Because the new version of a sigmoid function and its estimation has been covered in detail elsewhere (see Johansson, 1972b), only a

¹The reader is referred to Johansson (1972a), Chapter 4, for a more extensive discussion of the data. One point should be noted, however; although the focus here is upon national brands, the monthly expenditures on media advertising vary widely over time, so that in fact we have observations throughout the relevant range.

²Because the advertising data did not come from the surveys, the restriction to a 13 month time period did not apply and lags would thus not decrease the number of observations available.

a brief account will be given here. It is shown there that the model

$$(1) \quad \frac{y}{1-y} = ax_1^{b_1} x_2^{b_2} \cdots x_n^{b_n},$$

with y the dependent variable, and x_i , $i=1, \dots, n$, the independent variables, exhibits a sigmoid or S-shape whose skewness is determined by the estimated parameters b_i , $i=1, \dots, n$, and with a saturation level of 1.0. For a non-unit saturation level, say k , the model becomes

$$(2) \quad \frac{y}{k-y} = ax_1^{b_1} x_2^{b_2} \cdots x_n^{b_n}.$$

Finally, a non-zero intercept, say I , can be introduced by writing

$$(3) \quad \frac{y-I}{k-y} = ax_1^{b_1} x_2^{b_2} \cdots x_n^{b_n}.$$

The features of skewness of the S-shape, a non-unit saturation level, and a non-zero intercept are all desirable to allow for in estimating the advertising effect. As one model alternative, however, a completely symmetric sigmoid is also desirable. This is achieved by using a "loglinear" form of the above versions. Thus, the completely symmetric S-shape is depicted for model (3) by

$$(4) \quad \ln \frac{y-I}{k-y} = a + b_1 x_1 + b_2 x_2 + \cdots + b_n x_n,$$

\ln standing for the natural logarithm.

Possible estimation methods for the saturation level k and the intercept I could be derived from the approaches suggested by Croxton & Cowden, 1939, or Nelder, 1961, or Oliver, 1969, for different versions of the logistic function. The amount of computation involved, especially when a priori notions about k and I are rudimentary, is considerable, however. Another and much simpler approach which

suggested itself was to use the construction of the questionnaire in developing an estimate of the appropriate values. Thus, the upper limit on the proportion of purchasers of a particular brand was clearly the number of triers of the brand during the last six months. If all these triers purchased last time around then for that month the saturation level would in fact have been reached. Note that here the saturation level is allowed to vary between months, a feature which at first might seem undesirable but in fact has some advantages over a static level. For one, it tends to eliminate variations in observed proportions due simply to random variations between consecutive monthly samples. Second, it will in fact also allow for an actual change in the saturation level over time, a phenomenon that sometimes can be seen as desirable (see Nicosia, 1966).

For the intercept I the questionnaire provides another measure--the proportion of users having bought the brand last and the next to last time. Again, the measure is in fact the lower bound on the purchase proportion; and again a rationale behind the choice can be seen. As advertising goes towards zero, one would assume that some users would continue to buy, namely those that are brand loyal. Then the proportion repeating purchase can be seen as a proxy variable for such loyalty.

If we assume the saturation level k and the intercept I thus known, a natural procedure for estimating the parameters of these models would seem to be the ordinary least squares. Because of the limitation of the variation in y , however, this is not the correct procedure. The error variance will be a function of the value taken by y ; thus, there is a problem of heteroscedasticity. An approximate

expression for this error variance is developed in the case where y is a proportion (the case dealt with here) in Johansson, 1972b. Consequently, the procedure followed here consist of first dividing through each observation by the square root of this error variance after which ordinary least squares are applied (a special case of generalized least squares).

Further Model Specification

One logical way to see how well the generalized sigmoid fits the data is to test it against alternative functions. This was done by specifying a set of models directly explaining the sample proportions observed:

$$(10) \quad PUR_{it} = f_1 (TRIAL_{i,t-1}, PREF_{i,t-1}, DEAL_{it}, MAG_{it}, \dots, MAG_{i,t-j}, MAG_{NET_{it}}, \dots, MAG_{NET_{i,t-j}}, SPOT_{it}, \dots, SPOT_{i,t-j}, SPOT_{NEWS_{it}}, \dots, SPOT_{NEWS_{i,t-j}}, NEWS_{it}, \dots, NEWS_{i,t-j}, NEWS),$$

$i = 1, \dots, 4, \text{ and } t = 1, \dots, 12,$

with j subscripted by media representing the longest lag to be considered, and where the subscript i stands for brand, and t for month. The variable definitions and measurement are as follows:

PUR = purchase; the proportion of product users who indicated that they bought the brand last time (used as a proxy measure of market share)

TRIAL = trial; the proportion of users indicating that they had tried the brand within the last six months

PREF = preference; the proportion of users who said they liked the brand better than any other brand

DEAL = deal; the proportion of users who did not buy their last brand at regular price

MAG = magazine advertising share; the share computed relative to total magazine advertising by the four national brands

NET = network TV advertising share, computed similarly to MAG

SPOT = spot TV advertising share, computed similarly to MAG

NEWS = newspaper advertising share, computed similarly to MAG

This model was estimated using a linear form, a semi-logarithmic form and a double-logarithmic form. Each one of these three functions can be seen as an alternative to the S-curve.

The "odds" model was used to specify the competing S-curve. The following model

$$(11) \frac{PUR_{it}}{TRIAL_{it} - PUR_{it}} = f_2, (PREF_{i,t-1}, DEAL_{it}, MAG_{it}, \dots, MAG_{it-j}, NET_{it}, \dots, NET_{i,t-j}, SPOT_{it}, \dots, SPOT_{i,t-j}, SPOT, NEWS_{it}, \dots, NEWS_{i,t-j}, NEWS),$$

$i = 1, \dots, 4, \text{ and } t=1, \dots, 12,$

variables defined as above, was run as a loglinear and as a doublelog form, exhibiting the symmetric and skewed sigmoid, respectively.

Finally, the odds model with the intercept term included was run:

$$(12) \frac{PUR_{it} - REP_{it}}{TRIAL_{it} - PUR_{it}} = f_3, (PREF_{i,t-1}, DEAL_{it}, MAG_{it}, \dots, MAG_{i,t-j}, NET_{it}, \dots, NET_{i,t-j}, SPOT_{it}, \dots, SPOT_{i,t-j}, SPOT, NEWS_{it}, \dots, NEWS_{i,t-j}, NEWS),$$

$i=1, \dots, 4, \text{ and } t=1, \dots, 12,$

where REP = repeat; the proportion of product users who indicated that they bought the brand last time and the time before that. Again a loglinear and a doublelog form were introduced.

Some remarks on the specifications are in order. First, the absence of price (absolute as well as relative price) is motivated by the almost constant price levels maintained for the brands throughout the period. Some price competition took place in the form of specials, coupons, etc., which are picked up by the DEAL variable. TRIAL and PREF are introduced so as not to ascribe to advertising some effect that is due to experience and affect towards the brand from the past.³

Some potentially important variables are left out of the models. This includes product quality, distribution variables such as retail coverage and intensity, and the influence of closely related offerings under the same brand name. In general, these variables were assumed constant for the period in question, an assumption warranted by preliminary investigation. Not observed variables that could conceivably change during the period--in-store promotions other than deals, advertising not included, such as direct mail, for example--had to be seen as randomized, an undesirable approach but without alternatives.

There is also a question of causability: Does advertising cause purchases or do purchases cause advertising? Because of the short time intervals dealt with (months) we would argue that actual purchases cannot cause advertising. There is very little chance that feedback from the market can be received that fast, and then acted upon. On the other hand, one might visualize that expected purchases would affect advertising. As Zellner et.al. (1966) show, however, when the causal

³ When TRIAL was introduced as the saturation level, the TRIAL variable on the right hand side was eliminated for obvious reasons.

mechanism is non-deterministic this might not create any trouble; purchases regressed upon advertising is still the correct model.

Final Model Specification

The early regression runs were aimed at specifying the models more precisely. With the low number of observations available for each brand, it was deemed very desirable to pool the observations in some fashion. With the brands all being national and established, it seemed justifiable to assume the coefficients for the trial, deal, and preference variables to be very similar. With reference to the media advertising variables, however, the basic heterogeneity (due to creative, vehicle, and other within-media-differences) of the variables forestalled a parallel argument. Thus, the initial runs allowed for separate coefficients for each brand's media variables, but constrained the trial, deal, and preference coefficients to be the same for each brand.⁴ Although the low degrees of freedom made tests of the significance of the coefficients weak the results were remarkably consistent. The advertising coefficients were low and very similar across brands.

Because of the low number of observations (with a lagged dependent variable introduced, there were 12 data points per brand) only current media advertising (4 media variables) was included in these runs. In order to assess differences between brands for lagged advertising, all media advertising for the preceding month was summed and included as one additional variable. Again, no great brand differences emerged in terms of the advertising coefficients. Runs were made in parallel for

⁴ The approach used assigns dummy variables for each brand's separate slope coefficient; it is well discussed by Gujarati (1970).

advertising expenditures and for advertising shares (where the denominator consisted of the four brands' total advertising in the particular medium). No differences between brands obtained, although shares tended to do better (in terms of signs and significance levels of the coefficient estimates) than expenditures. Furthermore, running heavy and light users separately uncovered no significant differences (except for the intercept) between the two groups.

For the final runs, then, the four brands were pooled allowing only a separate intercept (but no separate slope coefficients) for each brand, in the usual analysis of covariance approach. With the lagging of the dependent variable one period, there were 12 observations per brand; with four brands pooled, there were 48 observations. In addition, the heavy and light users were pooled, again allowing for a separate intercept. Thus, the number of observations available for the final runs was 96. Advertising was measured as each brand's share of the four brand's total expenditures for the medium in question.⁵

In the initial "pooled" runs, advertising at t and $t-1$ (8 media variables) was introduced. The results generally showed no significant impact, and at times the effect seemed to be negative. The introduction of longer time lags was deemed necessary, and a Koyck distributed lag approach was attempted. The result was unsatisfactory, with no significant impact for the lagged dependent variable (which also ruled out an autoregressive model), and a more general lag structure seemed

⁵ Thus making it very possible that a brand's media input places it on the increasing part of the response curve.

needed. The one used was the polynomial distributed lag approach of Almon's (1965). A constrained quadratic form from t to t-4 showed by far the best fit and was the one used in the final runs. The constraints consisted of zero restrictions at t+1 and at t-5. With the constraints incorporated, each media variable introduced in fact represented a moving sum of the shares from time t to t-4. The weights of the sum were (in order) 1.0, 1.6, 1.8, 1.6, and 1.0. Accordingly, the advertising impact for each medium culminates at time t-2, with a smooth decline before and after that point.⁶

Results

As a first step in analyzing the results, it seems natural to determine which model is the "best". One attractive criterion for model choice is the predictive test, which necessitates observations in addition to those used for estimation. In this case, however, the initial model refinement did in fact draw upon the whole data base. As a consequence, short of generating new data, this avenue was closed. The best alternative is to ask how well the different models explain the variation observed in the available data. This leads to the use of the measure of the coefficient of determination, the R-square.

One problem in using the R^2 measure when choosing between alternative models which differ with respect to the form of the dependent variable, is that the relative explanation of a transformed variable is of less interest than the explanation of the variations in the original dependent variable. Thus, in our case, we are less interested

⁶ Although each medium was allowed to take on a different lag structure independent of other media, this structure turned out to be the best one for all the four media.

in how well the purchase odds, say, are explained, and more concerned about the explanation of the original sample proportion of purchasers. The transformation to odds is only "auxiliary" for estimation purposes. Similarly taking the logarithms of the dependent variable before regressing will give us an R^2 not immediately interpretable as explaining the relative amount of variation in the original dependent variable.

The solution to this problem lies in retransforming the "predicted" observations on the regression (from which the residuals are computed) back to a prediction of the original variables. Thus, if the dependent variable is "logged" before regression, antilogs are taken of the predicted observations. Similarly, for the odds and the odds with the intercept, the predicted values are transformed back to predictions of the original proportions, by solving (for the odds) $p/(k-p) = y$, for p with k given and y representing the predicted values from the regression run. Then the retransformed predicted data series is correlated with the original observations and a new "corrected" R^2 is derived.⁷

For the seven regression runs specified above, Table 1 gives the R-square in terms of explanation of the original sample proportions (the corrected R^2). All values are adjusted for degrees of freedom.

⁷ Note that when antilogs are taken, the predicted values are not really the expected values of the original observations: $E(\log x)$ is not equal to $\log E(x)$. Also, the assumption of a given k in retransforming the odds means that the actual observed k will have to be introduced, thus adding information to the odds explanation (and similarly for I in the odds with intercept model). However undesirable such features are in making the comparisons between the different models more difficult, the alternatives are worse (see Goldberger, 1964, p. 217).

One result indicated by these values is that the doublelog form does better than any other functional form, regardless of the measurement of the dependent variable. The differences are not great, but the consistency of the pattern indicates that they are fairly robust. Accordingly, the three doublelog models were tentatively selected as "the best". Although the R-square values differ widely between them, the additional information introduced in the two odds models makes a final choice of one model based upon this criterion alone somewhat arbitrary. It was decided to investigate them all further.

The regression coefficients and computed advertising impact, together with the beta coefficients from these three doublelog models are presented in Table 2. The individual monthly media coefficients are not presented--the total effect per medium is a summation of the impact over months t to t-4. Although some interesting hypotheses may be deduced from the different media coefficients, for our purposes here we will concentrate upon the total advertising impact. (The negative media coefficients for the odds and odds with intercept models are insignificant at the .05 level). Furthermore, the other-than-advertising variables, although interesting in themselves will not be discussed, except that it should be noted that, judging from the beta coefficients, the advertising is playing not just a marginal role in determining the variations in the purchase sample proportion.

So what do these results tell us about the applicability of the S-curve? First it should be noticed that the choice of the doublelog form over the loglinear function implies that the S-curve, if appropriate, will not be symmetric. Second, the impact of advertising in the proportions model is .33; as is well known, the doublelog function

with a coefficient between zero and one will have a positive but decreasing slope. According to this model then, response to advertising exhibits everywhere diminishing returns.⁸

But which one of the three "best" models should we accept as the one? Judging from the corrected R^2 values, can one say that the introduction of the intercept might be worth the effort, raising as it does the R^2 six percentage point? And what about the differences in R^2 between the odds model (.84) and the simpler proportions model (.63). Should we say that the difference is large enough that the added information introduced through the actual value of k when retransforming is more balanced? Even though before retransformating the proportions had a larger R^2 ? And is there any evidence in Table 2 that would favor the choice of one model over the other two?

It so turns out that a decision does not have to be made. For when the doublelog forms of the odds and the odds with intercept models are checked for their second partial derivatives with respect to advertising, these derivatives are negative in the relevant regions, indicating diminishing returns. To find the inflexion point for the odds doublelog model, we set⁹

$$(13) \quad \frac{b_i (k-2y)}{k} - 1 = 0,$$

that is,

$$(14) \quad b_i = \frac{k}{k - 2y} .$$

⁸ Notice that this type of response quite often is represented by the semilog function. The results in Table 1 indicate that perhaps the doublelog form, which is more flexible, is to be preferred.

⁹ See Johansson, 1972b.

In words, at the inflection point, the coefficient b_1 has to equal $k/(k-2y)$. With y always larger than zero, this means that b_1 has to be larger than one. From Table 2 we see that b_1 is less than one ($b_1 = .07$) indicating that the inflection point in fact occurs where y is negative. From (8) we see that the second partial derivative is, in fact, negative, implying diminishing returns in the relevant region.

For the odds with intercept model the same condition becomes

$$(15) \quad \frac{b_1(k-2y+I)}{k-I} - 1 = 0,$$

or

$$(16) \quad b_1 = \frac{k-I}{k-2y+I}.$$

Again, b_1 would have to be greater than one for the inflection point to fall in the relevant region (to see this, set y equal to its lower bound and solve for b_1). From Table 2 we see that the inflection point occurs at negative y -values, and again the second derivative is negative in the appropriate region. The S-curve does not correctly describe these advertising effects.

Concluding Comments

Through the research presented in this paper we found that advertising exhibits no sigmoid effect. The generality of this finding outside the given product class is not easy to assess. Further research is clearly needed on this score. Because the approach developed is very straightforward, there should be little problem in applying similar techniques to other sets of advertising data.

If the generality of the results cannot yet be assessed, do we know with certainty that the product class analyzed in fact does exhibit no sigmoid effect? Not quite, really. We eliminated the private

brands--they may encounter some threshold effect. Also, over longer time periods a threshold might in fact obtain. Indeed, it has to be pointed out that these two conditions together imply that we are dealing with quite small movements in the market shares of the different brands. In the final analysis this might be the reason for advertising not exhibiting any threshold stage in these data.

TABLE 1
THE COEFFICIENTS OF DETERMINATION

REGRESSAND	FUNCTIONAL FORM	ADJUSTED R ²
Purchase (Sample Proportion)	Linear	.58
	Semilog	.56
	Doublelog	.63
Purchase (Odds)	Loglinear	.83
	Doublelog	.84
Purchase (Odds With Intercept)	Loglinear	.88
	Doublelog	.90

TABLE 2

REGRESSION RESULTS FOR THE THREE DOUBLELOG MODELS

(Significance at the .05 level is indicated by a star)

Regressand Regressors	PURCHASE (SAMPLE PROPORTION)		PURCHASE (ODDS)		PURCHASE (ODDS WITH INTERCEPT)	
	Coefficient (Standard error)	Beta	Coefficient (Standard error)	Beta	Coefficient (Standard error)	Beta
Magazine Adv.	.04 (.05)	.29	.01 (.03)	.07	.002 (.03)	.01
Network TV Adv.	.11* (.05)	.32	-.04 (.04)	.12	-.03 (.05)	.08
Spot TV Adv.	.11* (.06)	.57	.05 (.05)	.24	.07 (.05)	.36
Newspaper Adv.	.07* (.03)	.47	.05* (.02)	.34	.08* (.02)	.50
ADVERTISING TOTAL	.33* (.07)		.07* (.03)		.12* (.04)	
DEAL _t	.37* (.09)	.54	.18 (.10)	.23	.31* (.10)	.41
TRIAL _{t-1}	-.26 (.17)	.27				
PREFERENCE _{t-1}	.21 (.11)	.29	.07 (.09)	.11	.03 (.11)	.06
CONSTANTS:						
Overall	1.53* (.35)		1.38 (1.73)		-5.08* (1.68)	
Brand 1	-.99 (.71)	.95	-4.39 (3.23)	.51	-4.21 (3.07)	.52
Brand 2	-.04 (.24)	.04	-.99 (1.55)	.01	-1.73 (1.62)	.21
Brand 3	-.19 (.23)	.18	-1.08 (1.66)	.13	-1.79 (1.72)	.22
Heavy/Light Users	-.37* (.11)	.41	-4.79* (.76)	.65	-2.07* (.86)	.30
Adjusted R ²	.56 (regressand) .63 (corrected)		.50 (regressand) .84 (corrected)		.30 (regressand) .90 (corrected)	
Durbin-Watson	1.94		1.94		2.10	
No. of Observations	96		96		96	

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